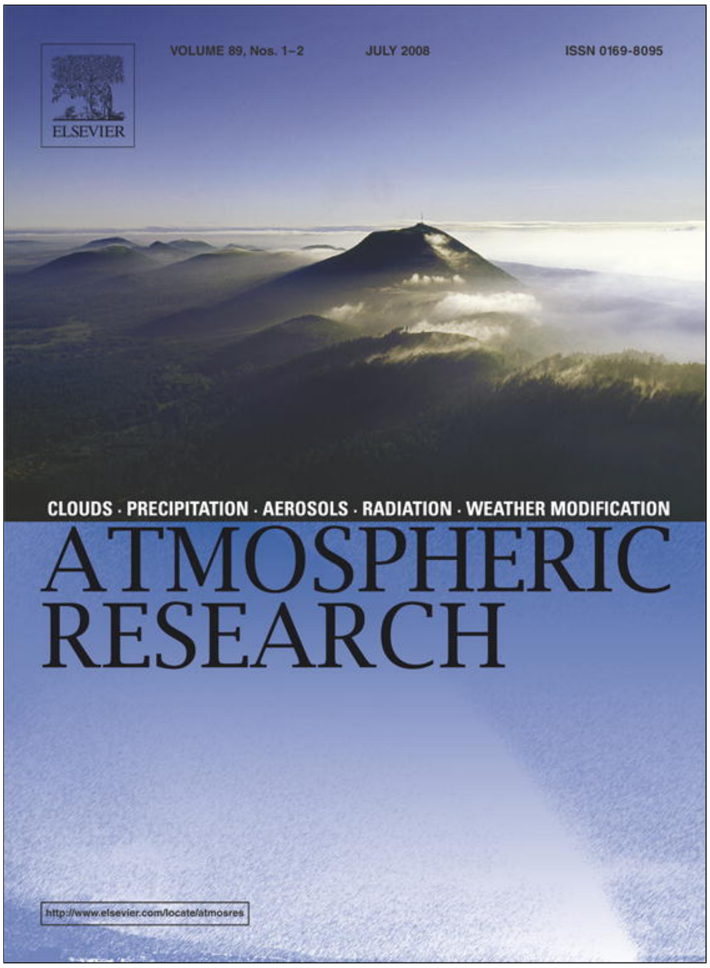
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Short communication

Phase functions of mixed clouds

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Abstract

A model of the phase function of a mixed cloud is proposed. It is based on Mie calculations for spherical water droplets and empirical phase function of a crystalline cloud measured *in situ*. The simple parameterization for the crystalline cloud phase function has been developed.

Keywords: Clouds; Light scattering

Mixed clouds are of a frequent occurrence and, therefore, it is of importance to understand how they scatter solar light. Such clouds are composed of water droplets and ice crystals. Simple approximations for extinction and absorption characteristics of mixed clouds are given elsewhere (Kokhanovsky, 2007). Here we propose a simple model, which reduces the calculation of the mixed cloud phase function to that of a water cloud, which can be derived from the Mie theory. The model is of importance for studies of light propagation and scattering in mixed clouds.

The phase function $p(\theta)$ of a mixed cloud gives the conditional probability for a photon to be scattered in a given direction specified by the scattering angle θ. It can be presented in the following form:

$$p(\theta) = \frac{K^w_{\text{sca}} p^w(\theta) + K^i_{\text{sca}} p^i(\theta)}{K^w_{\text{sca}} + K^i_{\text{sca}}}, \tag{1}$$

where K^w_{sca} is the scattering coefficient of water droplets and K^i_{sca} is the scattering coefficient of ice crystals, $p^w(\theta)$ and $p^i(\theta)$ are correspondent phase functions. Taking into account that it follows both for water and ice

$$K_{\text{sca}} = N\bar{C}_{\text{sca}}, \tag{2}$$

where $\bar{C}_{\text{sca}}$ is the average scattering cross section and N is the number of particles in a unit volume, and assuming that clouds do not absorb radiation and particles are randomly oriented, we obtain from Eq. (1):

$$p(\theta) = \frac{p^w(\theta) + \xi p^i(\theta)}{1+\xi} \tag{3}$$

where

$$\xi = \frac{N^i \Sigma^i}{N^w \Sigma^w}, \tag{4}$$

is the glaciation parameter and $\Sigma^{i(w)}$ is the average surface area of ice (water) scatterers and $N^{i(w)}$ is the correspondent number concentration. The glaciation parameter ξ is equal to the ratio of the total surface area of crystals in a mixed cloud to that of water droplets. We used here the asymptotic limit: $\bar{C}_{\text{sca}} = \bar{\Sigma}/2$ valid for nonabsorbing particles much larger than the wavelength

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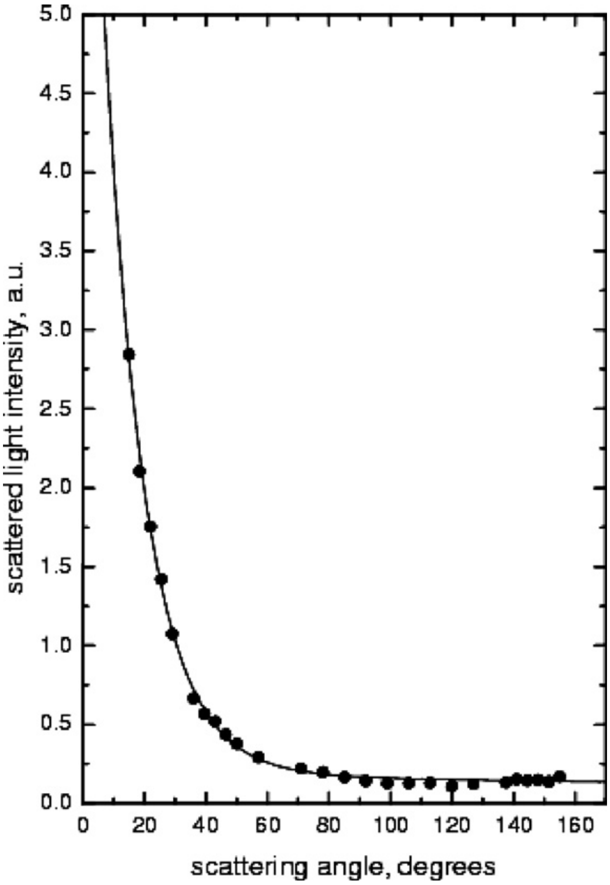

Fig. 1. *In situ* measured (Jourdan et al., 2003) scattered light intensity for an ice cloud (symbols) and a fit obtained using Eq. (9).

of incident light (Kokhanovsky, 2006). Such an assumption is always valid for crystals in the visible. For small droplets small corrections to Eq. (4) are needed (Kokhanovsky, 2007). They can be found from Mie theory. Eq. (4) can be also written in the following form:

$$\xi = \frac{C^i r^w_{\mathrm{ef}}}{C^w r^i_{\mathrm{ef}}}, \tag{5}$$

where $C^{i(w)}$ is the volumetric concentration of ice (water) in a cloud, $r^{i(w)}_{\mathrm{ef}}$ is the effective radius of ice (water) scatterers. The effective radius is defined as $r_{\mathrm{ef}}=3\bar{V}/\bar{\Sigma}$, where $\bar{V}$ is the average volume of particles in the unit volume of a cloud. The phase function of water clouds can be easily found from the Mie theory (Kokhanovsky, 2006). This is not the case for the phase function of ice crystals in the cloud. The various geometrical optics codes can be used to calculate the phase function of crystals. However, for this one needs to fix a shape of crystals and different shapes produce very different results with respect to $p^i(\theta)$. On the other hand, the shape of crystals in a cloud is not defined. Basically, ice clouds are composed of crystals of various shapes in different proportions. The same applies to snow on the ground.

Phase functions of such complex crystalline media cannot be calculated in a precise way. So we use the parameterization of the experimental measurements in crystalline clouds (Jourdan et al., 2003). The measured scattering diagram and also the approximation are shown in Fig. 1. The approximation has a following simple form:

$$I_{\mathrm{sca}}(\theta) = A\exp(-\alpha\theta) + B\exp(-\beta\theta), \tag{6}$$

where I_{sca} is the scattered light intensity (in arbitrary units), θ is the scattering angle in radians and $A=8.09$, $B=0.19$, $\alpha=4.3$, $\beta=0.11$. We propose to use Eq. (6) for the modeling of the phase function $p(\theta)$ of crystalline clouds. Namely, the phase function of nonabsorbing crystals can be presented as a sum of contributions due to diffraction of light on crystals $p^d(\theta)$ and the geometrical optics component $p^g(\theta)$, which, as we assume, is governed by Eq. (6). Then it follows:

$$p^i(\theta) = \frac{p^d(\theta) + p^g(\theta)}{2}, \tag{7}$$

where all phase functions shown in Eq. (7) are normalized as follows:

$$\frac{1}{2}\int_0^\pi p(\theta)\sin\theta d\theta = 1. \tag{8}$$

This enables to derive the following expression for the geometrical optics component taking into account Eq. (6):

$$p^g(\theta) = v\exp(-\alpha\theta) + q\exp(-\beta\theta), \tag{9}$$

where

$$v = \frac{2}{s(\alpha) + bs(\beta)}, q = \frac{2b}{s(\alpha) + bs(\beta)}, \tag{10}$$

$b=B/A\approx 0.023$ and

$$s(\xi) \equiv \int_0^\pi \exp(-\xi x)\sin x dx = \frac{1+\exp(-\pi\xi)}{1+\xi^2}. \tag{11}$$

Eq. (9) satisfies the normalization condition given by Eq. (8). We will use the fact that the random collection of irregularly shaped particles have the same angular structure of the diffraction peak as the collection of polydispersed spherical particles. Then we have (Kokhanovsky, 2006):

$$p^d(\theta) = \frac{4\langle J_1^2(k\theta r)\rangle}{\theta^2}, \tag{12}$$

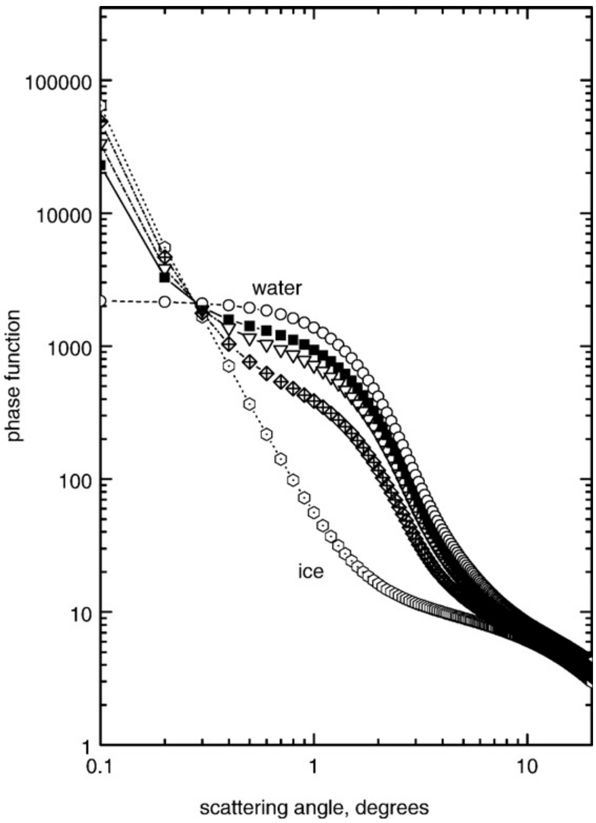

Fig. 2. The phase function of mixed clouds at ξ=0 (water cloud), ξ=0.5, 1.0, 3.0, and infinity(ice cloud). Further explanations are given in the text. Lower lines at 1°correspond to larger values of ξ.

where $k=2\pi/\lambda$, λ is the wavelength, r is the radius of particles, and angular brackets mean:

$$\langle J_1^2(k\theta r)\rangle = \frac{\int_0^\infty drr^2 f(r) J_1^2(k\theta r)}{\int_0^\infty drr^2 f(r)} \quad . \tag{13}$$

In particular, the following model of particle size distribution $f(r)$ for ice crystals can be used:

$$f(r) = r^6 \exp(-6r/r_0) \tag{14}$$

where r_0 is the mode radius.

The asymmetry parameter

$$g = \frac{1}{2}\int_0^\pi p(\theta)\sin\theta\cos\theta d\theta \tag{15}$$

is often used for the analysis of light transport in cloudy media. It follows from equations given above that

$$g = \frac{1+g_0}{2}, \tag{16}$$

where

$$g_0 = \frac{w(\alpha/2) + bw(\beta/2)}{s(\alpha) + bs(\beta)} \tag{17}$$

with

$$w(x) = \frac{1-\exp(-2\pi x)}{4(1+x^2)} \tag{18}$$

for the phase function given by Eq. (7). Calculations give: g_0=0.512 and g=0.756, which is close to measurements of g for ice clouds reported elsewhere (Garrett et al., 2001). Actually, it was found (Garrett et al., 2001) that g does not vary significantly for ice clouds. This points out to the fact that the phase function shown in Eq. (9) although obtained from a single measurement is representative for an ice cloud. This is mostly due to the fact that geometrical optical component does not dependent on the size of large nonabsorbing ice crystals. There is some dependence on the shape and we believe that it is well captured by Eq. (9). Further studies are needed to prove this assumption and check its validity on a statistical basis. However, we believe that for theoretical studies related to light propagation in mixed clouds, the model proposed is of a certain value.

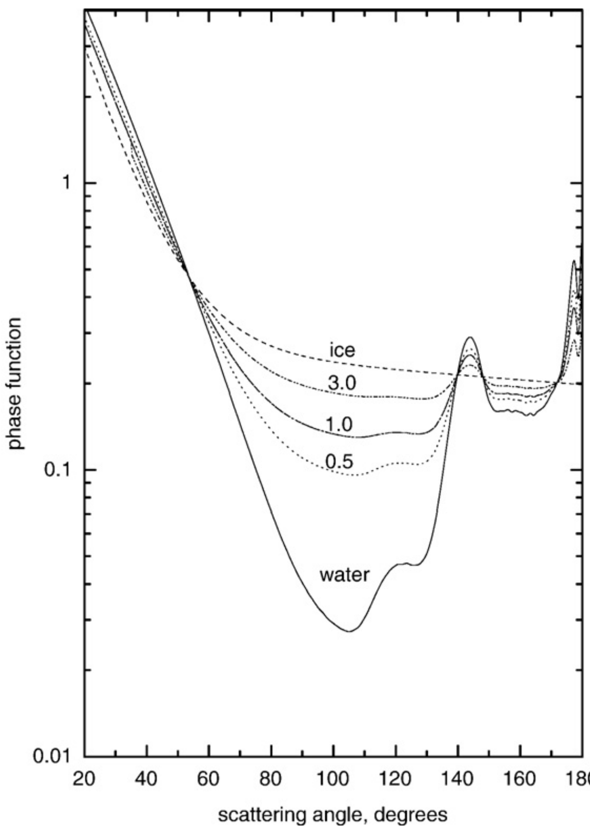

Fig. 3. The same as in Fig. 2 except for scattering angles larger than 20°. Numbers indicate the values of ξ.

In conclusion, we show the results of calculation of phase functions according to the described model for various values of the glaciation parameter ξ in Figs. 2, 3 at the wavelength 412 nm. The value of the effective radius $r_{ef} = 1.5r_0$ (see Eq. (14)) was assumed to be equal to 6 μm for water clouds and 100 μm for crystalline clouds. We conclude that the model presented here is capable to represent temporal changes of the phase function (e.g., during cloud freezing events). The phase function of ice clouds takes smaller values as compared to that of warm clouds in the range of scattering angles smaller than 0.3–10° for the case studied but rapidly increases as $\theta \to 0$ (see Fig. 2). It follows from Fig. 3 that the presence of ice crystals in the clouds dramatically change the light scattering patterns around 100° (side scattering). Correspondent changes can be detected if angular measurements of scattered light intensity are performed. It is interesting that rainbow and especially glory feature present for the case of mixed clouds even at comparatively large values of ξ (see Fig. 3). This is because the fact that light scattering by water droplets is more effective as compared to crystals in these scattering regions.

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